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132. Comparative Analysis of Methods for Evaluation of the Response of Large-Scale Systems to the Dynamic Loading

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1. Introduction

One of the most time and work consuming tasks in process of the analysis of large scale mechanical systems is to obtain the stress-strain state of elements of a system under the dynamic loading. There are several ways to solve the problem. In this paper two of them – the linear spectral method and the dynamic analysis method are described, analyzed and compared on the basis of the numerical analysis results obtained by using both methods in ANSYS5.6.3.

2. Linear spectral method

Linear spectral method is widely used in seismic analysis of elastic structures [1, 2, 3, 4]. The seismic loading is determined as the response spectra (mostly - the acceleration spectra) at every constrained node. This method also is suitable for evaluation of the response of large-scale systems to the combined seismic, force, inertial and thermal loading.

The linear spectral method is applied to the dynamic equations of a structure presented in modal coordinates. The analysis consists of the following steps:

- calculation of displacements and stresses of elements of a structure under the static loading that presents the normal operating conditions;
- the modal analysis the structure;
- the seismic loading corresponding to every mode is calculated on the base of the prescribed response spectra;
- the responses (displacements, moments, forces, etc.) corresponding to every mode of the considered structure are calculated by applying loading calculated in the previous step;
- the total response of the structure to the seismic loading is calculated where the contribution of the every mode is evaluated by using appropriate combining method (grouping method was used in this work);
- the results of calculations performed in previous step are added to the results of static analysis at the normal operating conditions and the strength of the mechanical system is estimated.

The matrix dynamic equation of an elastic structure reads as

$$\begin{bmatrix} [M_{NN}] & [M_{NK}] \\ [M_{KN}] & [M_{KK}] \end{bmatrix} \begin{Bmatrix} \{\ddot{U}_N\} \\ \{\ddot{U}_K\} \end{Bmatrix} + \begin{bmatrix} [C_{NN}] & [C_{NK}] \\ [C_{KN}] & [C_{KK}] \end{bmatrix} \begin{Bmatrix} \{\dot{U}_N\} \\ \{\dot{U}_K\} \end{Bmatrix} +$$

$$+ \begin{bmatrix} [K_{NN}] & [K_{NK}] \\ [K_{KN}] & [K_{KK}] \end{bmatrix} \begin{Bmatrix} \{U_N\} \\ \{U_K\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{R\} \end{Bmatrix}, \qquad (1)$$

where $\{U_N\}$, $\{U_K\}$ – vectors of nodal displacements of the structure, $[M]$, $[C]$ and $[K]$ - mass, damping and stiffness matrices, $\{R\}$ - reactions at nodes subjected to seismic excitation.

After transformations of equation (1) - decomposing the law of vibration of non-constrained nodes into two parts and making the proportional damping assumption $[C]=\beta[M]$, where β is a constant multiplier, equation (1) reads as

$$[M_{NN}]\{\ddot{U}_{Nd}\}+[C_{NN}]\{\dot{U}_{Nd}\}+[K_{NN}]\{U_{Nd}\}=[\hat{M}]\{\ddot{U}_K\}, \qquad (2)$$

where $[\hat{M}]=[M_{NN}][K_{NN}]^{-1}[K_{NK}]-[M_{NK}]$.

The natural frequencies of the structure ω_n are obtained by solving the equation

$$det([K_{NN}]-\omega^2[M_{NN}])=0, \qquad (3)$$

where n – number of the degrees of freedom of the structure.

When all modes are normalized in order to satisfy the relation $\{y_i\}^T[M_{NN}]\{y_i\}=1$, dynamic equation (2) splits into n independent equations:

$$\ddot{z}_i+2\omega_i\beta_i\dot{z}_i+\omega_i^2 z_i=\{y_i\}^T[\hat{M}]\{\ddot{U}_K\}, \quad i=1, 2, \ldots, n \qquad (4)$$

where $\{U_N\}=\{y_1\}z_1+\{y_2\}z_2+\cdots+\{y_n\}z_n$; $\{y_1\}$, $\{y_2\}$, …, $\{y_n\}$ - vectors of normalized natural forms; β_i – the damping factor of the i-th mode;

$$\{\ddot{U}_K\}=\{V\}a(t), \qquad (5)$$

where $a(t)$ – the prescribed time law of the acceleration of the support of the structure.

The equation of vibrations of the single modal d.o.f. corresponding to the natural frequency ω reads as

$$\ddot{x}+2\omega\beta\dot{x}+\omega^2 x=a(t). \qquad (6)$$

In accordance with equations (4), (5) and (6) the contribution of i-th mode to maximal displacements of the structure is

$$z_{imax}=\frac{A(\omega_i)}{\omega_i^2}\{y_i\}^T[\hat{M}]\{V\}, \qquad (7)$$

$$\{q_i\}=z_{imax}\{y_i\}. \qquad (8)$$

The contributions of all modal responses are combined as

$$q_j = \sqrt{\sum_{i=1}^{s} q_{ji}^2 + \sum_{r=1}^{p}\left\{\sum_{r=1}^{s_r}\left|q_{jr}\right|\right\}^2}\ , \qquad (9)$$

where q_j – the calculated displacement of j- degree of freedom of the structure, s- the number of natural frequencies values of which satisfy the inequality $(\omega_k - \omega_{k-1})/\omega_k > 0{,}1$, p – the number of groups of natural frequencies, satisfying the inequality $(\omega_k - \omega_{k-1})/\omega_k < 0{,}1$, s_r – the number of natural frequencies in every group.

When performing the calculations in ANSYS [5], maximal values of displacements have been combined by using the grouping method:

$$q_j = \sqrt{\sum_{k=1}^{n}\sum_{l=1}^{n} \varepsilon_{kl}\left|q_{jk} q_{jl}\right|}\ , \qquad (10)$$

where $\varepsilon_{ij} = \begin{cases} 1, (\omega_k - \omega_{k-1})/\omega_k \le 0{,}1 \\ 0, (\omega_k - \omega_{k-1})/\omega_k > 0{,}1 \end{cases}$

The resulting values of stresses, displacements, internal forces, velocities and accelerations are calculated as usually:

$$P_j = \sqrt{\sum_{k=1}^{3} P_{jk}^2}\ , \qquad (11)$$

where P_{jk} – the value of the parameter at the node (or in a finite element) j, obtained by performing calculations with respect to k direction of displacement of the support, P_j – the total value of this parameter.

The strength of the system and its elements is evaluated by comparing calculated values of the nodal displacements and stresses in elements with permissible ones.

3. The dynamic analysis method

The dynamic analysis method is based on the numerical integration of structural dynamic equations in time. In this case non-linearities of the structural behavior can be easily evaluated.

When analyzing mechanical system under the seismic loading, the curves of alteration of the seismic acceleration - accelerograms - on constrained nodes present the acting load. The response of structure to the such type of loading can be calculated on the basis of dynamic equation:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \left[\dot{M}\right]\{V\}a(t); \qquad (12)$$

where $\{U\}$- vector of nodal displacements of structure, $[M]$, $[C]$ and $[K]$ - mass, damping and stiffness matrices of structure, $\{V\}$ – vector of nodal masses of structure, $a(t)$ - the alteration of the seismic acceleration (accelerogram) of the foundation, carrying the structure to be analyzed (constrained nodes).

When performing dynamic analysis, the response of structure is calculated directly, by decomposing the accelerogram of dynamic loading into small steps, calculating the response of structure to every of them and adding every response to the sum of calculated before. The main deficiency of such a way for evaluation of response of the large-scale structures to dynamic loading is the large amount of information to be processed and time to perform that processing, because the response of the structure not only to one law of loading (moving of constrained nodes), but to several synthesized accelerograms must be calculated in this case, due to impossibility to use generalized response spectrums. To evaluate strength of structure the information about stress-strain state within every integration step must be taken into account. The only possible way to minimize the amount of work to be done - the considerable reduction of the number of degrees of freedom, however such a simplification of computational model makes negative influence upon the precision of analysis in most cases.

4. The comparison of the results of computations of response of the equipment of Ignalina NPP to the dynamic loading, obtained by using different methods of computations

The linear spectral method in engineering calculations is used for quite a long time, it is one of most widespread analytical adjacent methods. With improving and spreading discrete methods and computers this method is also used for investigating large-scale mechanical systems with the large number of degrees of freedom. But the results of such an analysis are to be considered only approximate, qualitative due to characteristic features of this method, for example, summing the response to static and dynamic loading in absolute values. In addition, this method may be used only for analyzing physically and geometrically linear elastic structures.

These principles can be confirmed by computations of drum-separator of Ignalina NPP (Fig. 1.) performed by using finite element analysis system ANSYS.5.6.3 [5]. The loading considered was the combination of normal operating conditions and maximum computational earthquake (6,5 points by Richter's scale).

The direct results of computations using linear spectral method are the fields of drum-separator nodal displacements, and after supplementary computations - corresponding fields of stresses. The displacement of fixing point of drum-separator's side support along the drum's longitudinal axis obtained in the way mentioned above was 61 mm, i.e. exceeds allowable value, specified in equipment documentation.

The fields of drum-separator nodal displacements and stresses in structure's elements are obtained by direct computations using dynamic analysis method. The curves of response displacements of drum-separator's side support are presented in fig. 2. It can be seen that maximum response displacement of drum-separator's side support fixing point along the drum's longitudinal axis is 14,7 mm, what is approximately 4 times smaller, than in case of computations using linear spectral method.

Noteworthy, that the first computation continued about 1,5 hour, the second one - approximately 150 hours, in addition, the second computation was performed on personal computer, with 4 times faster main processor and 4 times larger RAM comparatively to first computation. Therefore dynamic analysis method may be used to evaluate the response of large scale mechanical systems to dynamic loading only when having a possibility to use powerful enough hardware and software.

5. Conclusions

The linear spectral method suits better to reveal the weak elements of mechanical systems with large number of degrees of freedom, and for more deep and precise analysis of the fatigue, longevity of such systems, especially when analyzing nonlinear ones, dynamic analysis method is more acceptable. The qualities of dynamic analysis method overbalance the considerable amount of information to be processed and time to perform that processing in conjunction with necessity to use sufficient power hardware.

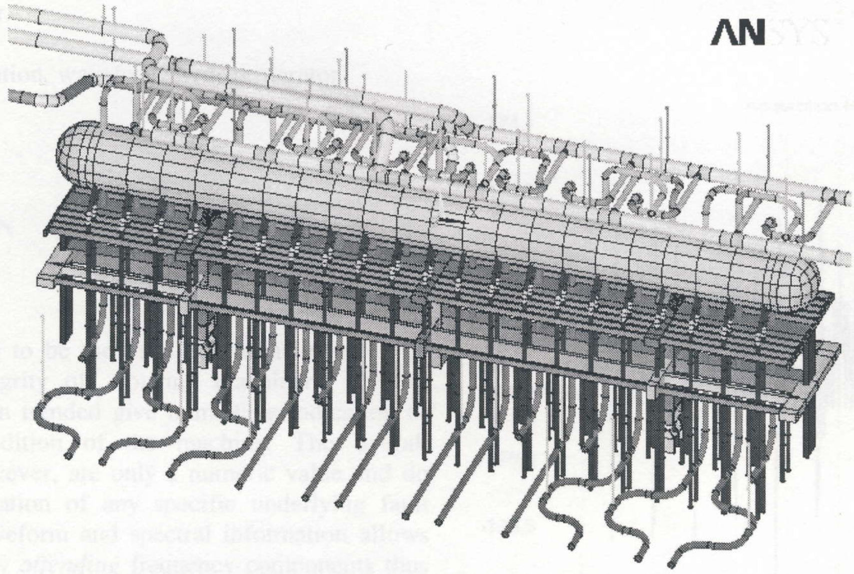

Fig. 1. The computational finite element model of drum-separator of Ignalina NPP

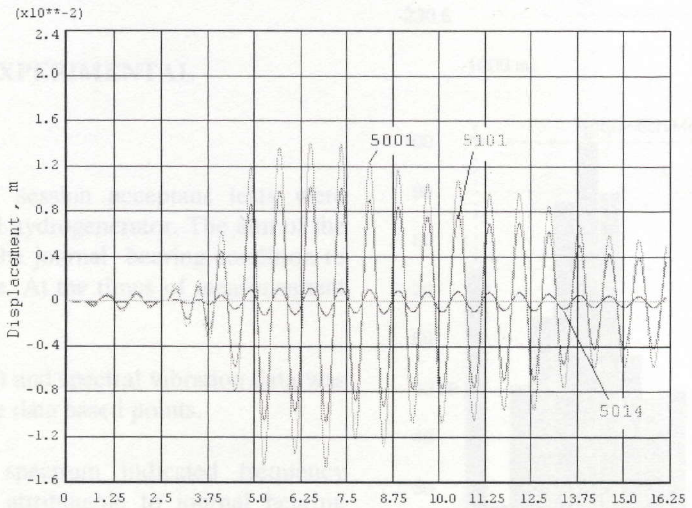

Fig. 2. The response curves - displacements of drum-separator side support nodes

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